Coincidences versus luminosity

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1 Analytic calculation of coincidence notes

This is basic analytical calculation of coincidence probability $\mathfrak{R}$ as a function of luminosity $\mathfrak{L}$. Coincidence here has a meaning of detecting anything in side A and C modules. The basic quantity which must be determined by simulation is the average number of tracks r_{TR} detected in BCM per proton-proton collision. Turning the focus to probability; for not detecting anything in side A in N proton-proton collisions probability can be written as:

$$\mathfrak{P}(0, r_{TR}NP_A) = e^{-r_{TR}NP_A}, \tag{1}$$

where $\mathfrak{P}(x, y)$ is Poisson distribution for x events at a average rate y and P_A stands for probability of detecting BCM track on side A (and analogue for C side). So

$$P_A = \frac{N_A}{N_A + N_C} \quad \text{and} \quad P_A + P_C = 1, \tag{2}$$

where N_i denotes the number of detected tracks on the corresponding side. For a coincidence we must have something on both sides so the appropriate expression is:

$$p_{A\wedge C} = \left(1 - e^{-P_A N r_{TR}}\right)\left(1 - e^{-(1-P_A)N r_{TR}}\right). \tag{3}$$

The above expression must be combined with the probability for N proton-proton interactions at given luminosity. This is also assumed to be Poissonian. The final expression, in terms of luminosity, is therefore written as

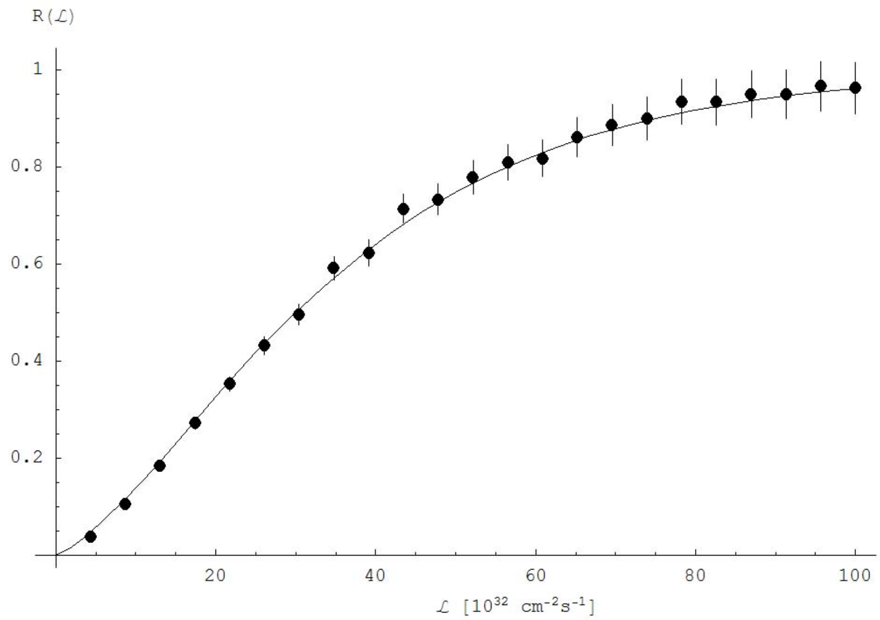
$$\mathfrak{R}(\mathfrak{L}) = \sum_{N=0}^{\infty} \mathfrak{P}\left(N, \frac{\mathfrak{L}}{\mathfrak{L}_\mathrm{o}}\right)\left[\left(1 - e^{-P_A N r_{TR}}\right)\left(1 - e^{-(1-P_A)N r_{TR}}\right)\right], \tag{4}$$

where $\mathfrak{L}_\mathrm{o}$ denotes luminosity at which on average one proton-proton collision is expected. In the region of low luminosities the above formula can be expanded and with dominating $N = 1$:

$$\mathfrak{R}(\mathfrak{L}) \simeq \left(1 - e^{-P_A r_{TR}}\right)\left(1 - e^{-(1-P_A) r_{TR}}\right)\frac{\mathfrak{L}}{\mathfrak{L}_\mathrm{o}} + O(\mathfrak{L}^2) \tag{5}$$

which shows a linear regime at low luminosities.

Simulation on 8000 proton-proton collision was done and coincidences were counted for various luminosities. This data, with $r_{TR} = 0.375$ also obtained from simulation, is plotted against prediction (4).



Slika 1: Prediction (continuous line) and simulation results (dots) are compared. In the error bars only statistical error of coincidence counting has been included and no r_{TR} uncertainty.

2 Effect of varying vertex position

Simulated results presented above were made for symmetrically positioned modules – the vertex was in the centre ($z = 0$). The movement of vertex has an impact on r_{TR} and on P_A since any movement means that the vertex is closer to A than to C side or vice versa. The following calculation is aiming to predict these dependences to the lowest order. All is done for the movements in the z direction.

Moving the vertex has two effects: the flux changes and the effective area of BCM changes due to slightly different relative positions of vertex and the module. I denote j as the flux in angle $d\Omega$ and Ω as BCM solid angle. Then the change in the number of detected tracks N, in one module, on dz movement of the module is expressed as:

$$\frac{dN}{dz} = \frac{dj}{dz}\Omega + j\frac{d\Omega}{dz}. \tag{6}$$

The following notation is used: z is the longitudinal coordinate of the module, transverse coordinate is denoted h, the area of active volume (diamond) is S_0 and the tilt angle of diamond relative to z axis if ϕ. All are assumed to be positive. From this the effective area S_{eff} of diamond, as seen by the vertex, can be calculated[1]:

$$S_{eff} = S_0 \frac{z sin\phi - h\cos\phi}{\sqrt{h^2 + z^2}}. \tag{7}$$

The derivative with respect to z is:

$$\frac{dS_{eff}}{dz} = S_0 \frac{h}{(h^2 + z^2)^{3/2}} \left(z\cos\phi + h\sin\phi\right) \tag{8}$$

[1] The following expression is valid for $h/z < \tan\phi$. To overstep this boundary the vertex displacement would have to be huge and the presented calculation is not valid anyway.

and is positive as should be, since making z larger means more perpendicular tracks from the vertex. We are interested in change of the solid angle, $\Omega = S(z)/r^2$ where $r = \sqrt{h^2+z^2}$, so:

$$\frac{d\Omega}{dz} = \frac{dS_{eff}}{dz}\frac{1}{r^2} - \frac{2S_{eff}}{r^3}\frac{dr}{dz}. \tag{9}$$

Since $dr/dz = z/r$ we can write:

$$\frac{d\Omega}{dz} = \frac{S_0}{(h^2+z^2)^{5/2}}\left(3hz\cos\phi - (2z^2-h^2)\sin\phi\right), \tag{10}$$

which has two terms. This is logical, since moving module in +z direction means: increasing distance form the vertex and rotating it in the track frame. The above expression describes geometrical changes. To compute the second term in (6) the flux j is also needed. The available information is charged particle density over η. I denote this N_η. From definition: the current is

$$j = \frac{dN}{d\Omega} = \frac{dN}{d\eta}\frac{d\eta}{d\theta}\frac{d\theta}{d\Omega} = \frac{1}{2\pi}N_\eta\left(1+\frac{h^2}{z^2}\right). \tag{11}$$

With standard definition of pseudorapidity η and θ being angle between z axes an vertex-module direction, we have:

$$\frac{d\theta}{dz} = -\frac{\sin^2\theta}{h} \quad \text{and} \quad \frac{d\eta}{d\theta} = -\frac{1}{\sin\theta}.$$

The equation (11) can be put to the test. If we multiply it by BCM solid angle for all eight modules 8Ω, we should obtain r'_{TR}. The predicted value is $r'^{P}_{TR} = 0.1958$ and simulation gives $r'_{TR} = 0.1919 \pm 0.0027$. Here must be stressed that r'_{TR} represents only the track rate of primary particles.

The next step are flux changes:

$$\begin{aligned}\frac{dj}{dz} &= \frac{dN_\eta}{dz}\frac{1}{2\pi\sin^2\theta} + N_\eta\frac{d}{dz}\left(\frac{1}{2\pi\sin^2\theta}\right) \\ &= \frac{1}{\pi h^2}\left(\left(\frac{dN_\eta}{d\eta}\right)\frac{\sqrt{h^2+z^2}}{2} + N_\eta z\right).\end{aligned} \tag{12}$$

Both derivatives, geometrical and flux, are now known and we can insert it in (6). We get,

$$\begin{aligned}\frac{dN}{dz} &= \frac{1}{\pi}\frac{S_0}{h^2(h^2+z^2)^{3/2}}\left[(z\sin\phi - h\cos\phi)\left(\left(\frac{dN_\eta}{d\eta}\right)\frac{\sqrt{h^2+z^2}}{2} + N_\eta z\right) + \right. \\ &\quad \left. + \frac{1}{2}N_\eta\left(3hz\cos\phi - (2z^2-h^2)\sin\phi\right)\right] = \frac{1}{4}\frac{dN_C}{dz},\end{aligned} \tag{13}$$

where N_C is the number of detected tracks on C side. To evaluate above expresion, the $N_\eta = 7.3$ and $\left(\frac{dN_\eta}{d\eta}\right) = -1.1$ were read from ATLAS TDR 14. From this $\frac{dP'_A}{dz}$ can be calculated:

$$\frac{dP'_A}{dz} = \frac{1}{N'_A+N'_C}\frac{dN'_A}{dz} = \frac{1}{r'_{TR}}\frac{dN'_A}{dz}, \tag{14}$$

where all quantities are defined only for primary tracks.

The final analytical estimation is therefore $\frac{dP'_A}{dz} = 0.0324/m$ and $\frac{dr'_{TR}}{dz} = 0$, since this is only the lowest order.

3 Simulation results

Simulation was done on 20k p-p collisions with vertex at $z = 0$ and 6k p-p collisions with $z = 10\,\mathrm{cm}$. The estimation is:

$$\frac{dP_A}{dz} = (-0.209 \pm 0.128)\,\frac{1}{m}, \tag{15}$$

and

$$\frac{dr_{TR}}{dz} = (0.11 \pm 0.12)\,\frac{1}{m}. \tag{16}$$

The track rate is still consistent with zero, but the change of P_A is very different from analytical prediction. Reason is that the prediction only includes particles that originate in vertex. These represent only half of particles that are detected by BCM ($r'_{TR} = 0.1919$, while $r_{TR} = 0.3983$ for all particles). The secondary particles therefore have much larger and negative contribution to $\frac{dP_A}{dz}$. This can be understood from figure 2. The origin of detected particles is plotted in z-r diagram. The left picture is for vertex at $z = 0$ and right, on much smaller statistics, for $z = 1m$. This is unreasonably large but it shows what happens: Pixel move out of vertex-module way and their contribution decreases on A side, while on C side the increase of Pixel-particles can be seen. This explains the negative sign of $\frac{dP_A}{dz}$.